

# Deterministic photonic spatial-polarization hyper-controlled-not gate assisted by quantum dot inside one-side optical microcavity

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Up to now, all the works about constructing quantum logic gates, an essential part in quantum computing, are focused on operating on one degree of freedom (DOF) of quantum systems. Here, we investigate the possibility to achieve scalable photonic quantum computing based on two DOFs of quantum systems and construct a deterministic hyper-controlled-not (hyper-CNOT) gate operating on both spatial-mode and polarization DOFs of a photon pair simultaneously, by using the giant optical Faraday rotation induced by a single-electron spin in a quantum dot inside a one-side optical microcavity as a result of cavity QED. With this hyper-CNOT gate and linear optical elements, two-photon four-qubit cluster entangled states can be prepared and analyzed, which gives an application to manipulate more information with less resources. We analyze the experimental feasibility of this hyper-CNOT gate and show that it can be implemented with current technology.

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## I. INTRODUCTION

Quantum mechanics theory, the core of quantum information, can largely improve the power in dealing and transmitting information. Quantum computing [1] is required in quantum information process for precise control and manipulation of the states of quantum systems. It is proven that two-qubit controlled-not (CNOT) gates (or the equivalent two-qubit quantum gates) assisted by single-qubit gates are sufficient for universal quantum computing, so it is important to construct two-qubit CNOT gate. By far, many works have been done on constructing two-qubit CNOT gate or its equivalent in notably trapped ions [2], nuclear magnetic spins [3], free electrons [4], and polarized photons [5–10]. The pioneering work by Knill, Laflamme, and Milburn (KLM) [5] showed that a photonic CNOT gate could be created with the maximal success probability of  $3/4$ , resorting to only single-photon sources, detectors, and linear optical elements such as beam splitters. Since this original work, there has been a significant progress [6–9] in creating CNOT gate with linear optics, which realizes probabilistically the nonlinear coupling between two photons by using interference, at least two ancilla photons, single-photon detectors, and conditioning. With nonlinear optics, a deterministic CNOT gate can also be constructed. In 2004, Nemoto and Munro [10] constructed a near deterministic CNOT gate using several single photon sources, linear-optical elements, photon number resolving quantum nondemolition detectors, and feed-forward. The key element of the CNOT gate in their proposal is the quantum nondemolition detector (QND) based on cross-Kerr nonlinearity.

Although there are many valuable works on construct-

ing quantum logical gates, especially CNOT gate, they are all focused on operating on one degree of freedom (DOF) of quantum systems. There are, by far, no quantum entangling gates operating on more than one DOF of quantum systems. In fact, there are many advantages for dealing with quantum information process in a larger Hilbert space, especially for its robustness against noise [11] and the high channel capacity. High-dimensional entanglement has been realized in multipartite [12, 13] and multidimensional [14, 15] quantum systems. Although a universal quantum computing can be realized with two-qubit CNOT gates and single-qubit operations, it will be convenient to have multi-qubit quantum logic gates in quantum computation. In 2006, Fiurášek [16] proposed some schemes for the direct probabilistic realization of the fundamental Toffoli and Fredkin gates with linear optics, and he presented a scheme for linear optical quantum Fredkin gate based on the combination of experimentally demonstrated linear optical partial-SWAP gate and controlled-Z gates in 2008 [17]. Gong *et al* [18] also discussed the realization of a quantum Fredkin gate by using CNOT gates with only linear optics and single photons. The large Hilbert space with more than one degree of freedom (DOF) has also been discussed in some applications in quantum communication in the past few years [19–24], especially for hyperentanglement which is defined as quantum systems entangled in more than one DOF. Besides the task in which hyperentanglement is used to assist quantum information processing in one DOF, the complete analyzer for hyperentangled Bell states has also been constructed [25] with cross-kerr nonlinearity to increase the channel capacity of long-distance quantum communication in more than one DOF.

In this paper, we investigate the possibility to achieve scalable photonic quantum computing based on two DOFs of quantum systems, different from all the existing works about constructing quantum logic gates as the latter are focused on operating on one DOF of quantum sys-

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tems. We construct a deterministic hyper-controlled-not (hyper-CNOT) gate operating on both spatial-mode and polarization DOFs of a photon pair simultaneously, assisted by two one-side optical microcavities (QD-cavity). Exploiting the giant optical Faraday rotation induced by a single-electron spin in a QD-cavity system on the left-circularly and the right-circularly polarized lights, we construct a four-qubit controlled-Z gate by using the two electron spins as the control qubits and the polarization and the spatial modes of a photon as the two target qubits, respectively. After the second photon interacts with the two QD-cavity systems, the two electron spins are detected and a deterministic hyper-CNOT gate could be constructed with single-qubit operations. As an application of this hyper-CNOT gate, one can use it to prepare two-photon four-qubit cluster entangled states easily and analyze the 16 cluster states simply, resorting to some linear optical elements. We analyze the experimental feasibility of this hyper-CNOT gate and our result shows that it can be implemented with current technology.

## II. CONSTRUCTION OF DETERMINISTIC SPATIAL-POLARIZATION HYPER-CONTROLLED-NOT GATE

In 2008, Hu *et al* [26, 27] pointed out that the interaction of left-circularly and right-circularly polarized lights with a QD-cavity system can be used in quantum information process. With this optical property of QD-cavity systems, multi-qubit entangler [26–28] and photonic polarization Bell-state analyzer [29, 30] can be constructed. In 2010, Bonato *et al* [30] constructed a CNOT gate on hybrid quantum system composed of a photon and an electron spin using the interface between the photon and electron spin in double-sided QD-cavity system in the weak coupling regime.

The QD-cavity system used in our proposal is constructed by a singly charged QD (e.g., a self-assembled In(Ga)As QD or a GaAs interface QD) located in the center of a one-side optical resonant cavity to achieve the maximal light-matter coupling. According to Pauli's exclusion principle, a negatively charged exciton ( $X^-$ ) consisted of two electrons bound to one hole [31] can be optically excited when an excess electron is injected into the QD. The optical resonance of  $X^-$  with circularly polarized lights depends on the excess electron spin in the QD [32], shown in Fig.1. For the excess electron-spin state  $|\uparrow\rangle$ , the negatively charged exciton  $|\uparrow\downarrow\uparrow\rangle$  with two electron spins antiparallel is created by resonantly absorbing a left-circularly polarized light  $|L\rangle$ . Here  $|\uparrow\rangle$  describes the heavy-hole spin state  $|+\frac{3}{2}\rangle$ . For the excess electron spin  $|\downarrow\rangle$ , the other negatively charged exciton  $|\downarrow\uparrow\downarrow\rangle$  can be created by resonantly absorbing a right-circularly polarized light  $|R\rangle$ . Here  $|\downarrow\rangle$  describes the heavy-hole spin state  $|-\frac{3}{2}\rangle$ . This optical process can be described by Heisenberg equations for the cavity field

operator  $\hat{a}$  and  $X^-$  dipole operator  $\hat{\sigma}_-$  in the interaction picture [33],

$$\begin{aligned}\frac{d\hat{a}}{dt} &= -[i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}] \hat{a} - g \hat{\sigma}_- - \sqrt{\kappa} \hat{a}_{in}, \\ \frac{d\hat{\sigma}_-}{dt} &= -[i(\omega_{X^-} - \omega) + \frac{\gamma}{2}] \hat{\sigma}_- - g \hat{\sigma}_z \hat{a}, \\ \hat{a}_{out} &= \hat{a}_{in} + \sqrt{\kappa} \hat{a},\end{aligned}\quad (1)$$

where  $g$  represents the coupling strength between the cavity mode and  $X^-$ .  $\kappa/2$  and  $\kappa_s/2$  represent the decay rate and the side leakage rate of the cavity field, respectively.  $\gamma/2$  represents the decay rate of  $X^-$ .  $\omega_{X^-}$ ,  $\omega$ , and  $\omega_c$  represent the frequencies of the  $X^-$  transition, the input probe light, and the cavity mode, respectively.

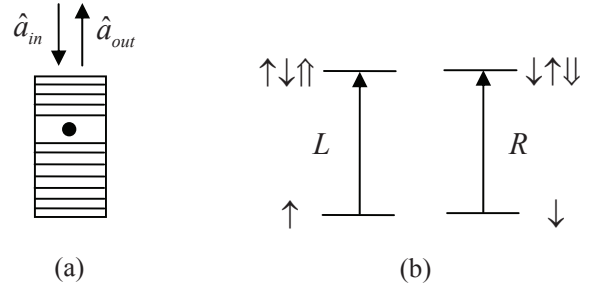


FIG. 1: The  $X^-$  spin-dependent transitions with circularly polarized lights. (a) A charged QD inside a one-side micropillar microcavity interacting with circularly polarized lights. (b)  $X^-$  spin-dependent optical transition rules due to the Pauli's exclusion principle.  $L$  and  $R$  represent a left-circularly and a right-circularly polarized lights, respectively.  $\uparrow$  and  $\downarrow$  represent excess electron spin.  $\uparrow\downarrow\uparrow$  and  $\downarrow\uparrow\downarrow$  represent negatively charged exciton  $X^-$ .

If  $X^-$  stays in the ground state at the most time (i.e.,  $\langle\sigma_z\rangle = -1$ ), the reflection coefficient of QD-cavity system for this weak excitation condition is [26]

$$r(\omega) = 1 - \frac{\kappa[i(\omega_{X^-} - \omega) + \frac{\gamma}{2}]}{[i(\omega_{X^-} - \omega) + \frac{\gamma}{2}][i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}] + g^2}. \quad (2)$$

In the condition  $g = 0$ , the QD is uncoupled to the cavity (the cold cavity), and the reflection coefficient becomes

$$r_0(\omega) = \frac{i(\omega_c - \omega) - \frac{\kappa}{2} + \frac{\kappa_s}{2}}{i(\omega_c - \omega) + \frac{\kappa}{2} + \frac{\kappa_s}{2}}. \quad (3)$$

Now, the optical process based on the  $X^-$  spin-dependent transitions is obtained. For a light in the state  $|L\rangle$ , a phase shift of  $\varphi_h$  is gotten for a hot cavity (the QD is coupled to the cavity) with the excess electron spin in the state  $|\uparrow\rangle$ , and a phase shift of  $\varphi_0$  is gotten for a cold cavity with the excess electron spin in  $|\downarrow\rangle$ . For a light in the state  $|R\rangle$ , a phase shift of  $\varphi_0$  is gotten for a cold cavity with the excess electron spin in  $|\uparrow\rangle$ , and a phase shift of  $\varphi_h$  is gotten for a hot cavity with the excess electron

spin in  $|\downarrow\rangle$ . By adjusting the frequencies of the light ( $\omega$ ) and the cavity mode ( $\omega_c$ ), the reflection coefficient can reach  $|r_0(\omega)| \cong 1$  and  $|r_h(\omega)| \cong 1$  when the cavity side leakage is negligible. If the linearly polarized probe beam in the state  $\frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$  is put into a one-side QD-cavity system in the superposition spin state  $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ , the state of system composed by the light and the electron spin after reflection is

$$\frac{1}{2}(|R\rangle + |L\rangle) \otimes (|\uparrow\rangle + |\downarrow\rangle) \rightarrow \frac{1}{2}e^{i\varphi_0}[(|R\rangle + e^{i\Delta\varphi}|L\rangle)|\uparrow\rangle + (e^{i\Delta\varphi}|R\rangle + |L\rangle)|\downarrow\rangle], \quad (4)$$

where  $\Delta\varphi = \varphi_h - \varphi_0$ ,  $\varphi_0 = \arg[r_0(\omega)]$ , and  $\varphi_h = \arg[r_h(\omega)]$ .  $\theta_F^\uparrow = (\varphi_0 - \varphi_h)/2 = -\theta_F^\downarrow$  is Faraday rotation angle. After reflection, the light and the spin become entangled with the different phase shifts of  $|L\rangle$  and  $|R\rangle$  lights.

The principle of our deterministic spatial-polarization hyper-CNOT gate for a two-photon system in two DOFs with the optical property of one-side QD-cavity systems is shown in Fig.2. The QD<sub>1</sub> and QD<sub>2</sub> are used to operate on the spatial mode and the polarization DOFs, respectively. By adjusting the frequencies  $\omega - \omega_c \approx \kappa/2$  to get the phase shifts of the left and the right circularly polarized lights as  $\varphi_0 = -\pi/2$  and  $\varphi_h = 0$ , the interaction of a single photon with a QD-cavity system can be described as

$$\begin{aligned} |R, \uparrow\rangle &\rightarrow -i|R, \uparrow\rangle, & |L, \uparrow\rangle &\rightarrow |L, \uparrow\rangle, \\ |R, \downarrow\rangle &\rightarrow |R, \downarrow\rangle, & |L, \downarrow\rangle &\rightarrow -i|L, \downarrow\rangle. \end{aligned} \quad (5)$$

We assume the initial states of the excess electron spin in QD<sub>1</sub> and the photon  $a$  are  $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_1}$  and  $|\Phi_a\rangle_0 \equiv (\alpha_1|R\rangle + \alpha_2|L\rangle)_a \otimes (\gamma_1|a_1\rangle + \gamma_2|a_2\rangle)$ , respectively. Here  $|a_1\rangle$  and  $|a_2\rangle$  represent the two spatial modes of the photon  $a$ . After the photon  $a$  passes through CPBS and HWP<sub>1</sub>, interacts with the electron spin in QD<sub>1</sub>, and then passes through another HWP<sub>1</sub>, CPBS and WP<sub>1</sub> in Fig.2, the state of the system composed of QD<sub>1</sub> and the photon  $a$  is changed from  $|\Phi_{ae_1}\rangle_0$  to  $|\Phi_{ae_1}\rangle_1$ . Here

$$\begin{aligned} |\Phi_{ae_1}\rangle_0 &= (\alpha_1|R\rangle + \alpha_2|L\rangle)_a \otimes (\gamma_1|a_1\rangle + \gamma_2|a_2\rangle) \\ &\quad \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_1}, \\ |\Phi_{ae_1}\rangle_1 &= \frac{1}{\sqrt{2}}(\alpha_1|R\rangle + \alpha_2|L\rangle)_a \otimes [|\uparrow\rangle_{e_1}(\gamma_1|a_1\rangle \\ &\quad + \gamma_2|a_2\rangle) + |\downarrow\rangle_{e_1}(\gamma_1|a_1\rangle - \gamma_2|a_2\rangle)]. \end{aligned} \quad (6)$$

This is the result of the controlled-Z gate constructed by using the electron spin  $e_1$  as the control qubit and the spatial modes of the photon  $a$  as the target qubit.

The initial state of QD<sub>2</sub> in Fig.2 is prepared to be  $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_2}$ , and the frequencies of the input photon and the cavity mode are adjusted to be  $\omega - \omega_c \approx \kappa/2$ , as the same as those for QD<sub>1</sub>. After the photon  $a$  interacts with QD<sub>2</sub> and passes through WP<sub>2</sub> as shown in Fig.2, the

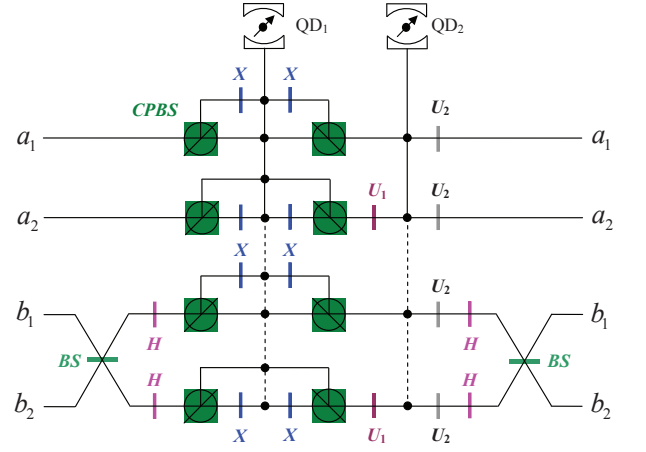


FIG. 2: (Color online) Schematic diagram of our spatial-polarization hyper-controlled-not gate operating on both the spatial-mode and polarization degrees of freedom of a two-photon system. BS represents a 50:50 beam splitter which is used to perform a Hadamard operation on the spatial-mode states of a photon. CPBS represents a polarizing beam splitter in the circular basis, which transmits the photon in the right-circularly polarization  $|R\rangle$  and reflects the photon in the left-circularly polarization  $|L\rangle$ , respectively.  $X$  represents a half-wave plate (HWP<sub>1</sub>) which is used to perform a polarization bit-flip operation  $X = |R\rangle\langle L| + |L\rangle\langle R|$ .  $H$  represents a half-wave plate (HWP<sub>2</sub>) which is used to perform a polarization Hadamard operation  $[|R\rangle \leftrightarrow \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)]$  and  $[|L\rangle \leftrightarrow \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle)]$ .  $U_1$  represents a wave plate (WP<sub>1</sub>) which is used to perform a polarization phase-flip operation  $U_1 = -i|R\rangle\langle R| - i|L\rangle\langle L|$ .  $U_2$  represents a wave plate (WP<sub>2</sub>) which is used to perform a polarization phase-flip operation  $U_2 = |R\rangle\langle R| - i|L\rangle\langle L|$ .

state of the complicated system composed of QD<sub>1</sub>, QD<sub>2</sub>, and the photon  $a$  is changed from  $|\Phi_{ae_1e_2}\rangle_1$  to  $|\Phi_{ae_1e_2}\rangle_2$ . Here

$$\begin{aligned} |\Phi_{ae_1e_2}\rangle_1 &= |\Phi_{ae_1}\rangle_1 \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)_{e_2}, \\ |\Phi_{ae_1e_2}\rangle_2 &= \frac{1}{2}[|\uparrow\rangle_{e_2}(\alpha_1|R\rangle + \alpha_2|L\rangle)_a + |\downarrow\rangle_{e_2}(\alpha_1|R\rangle \\ &\quad - \alpha_2|L\rangle)_a] \otimes [|\uparrow\rangle_{e_1}(\gamma_1|a_1\rangle + \gamma_2|a_2\rangle) \\ &\quad + |\downarrow\rangle_{e_1}(\gamma_1|a_1\rangle - \gamma_2|a_2\rangle)]. \end{aligned} \quad (7)$$

This is the result of the four-qubit controlled-Z gate constructed by using the two electron spins as the control qubits and the polarization and the spatial modes of the photon  $a$  as the two target qubits. That is, when the spin of the electron  $e_1$  is in the state  $|\downarrow\rangle_{e_1}$ , the spatial mode  $|a_2\rangle$  of the photon  $a$  obtains a phase shift  $\pi$ , and when the spin of the electron  $e_2$  is in the state  $|\downarrow\rangle_{e_2}$ , the polarization mode  $|L\rangle$  of the photon  $a$  obtains a phase shift  $\pi$ .

The second photon  $b$  is prepared in the initial state  $|\Phi_b\rangle_0 \equiv (\beta_1|R\rangle + \beta_2|L\rangle)_b \otimes (\delta_1|b_1\rangle + \delta_2|b_2\rangle)$ . After Hadamard operations are performed on both the spatial mode and the polarization DOFs of the photon  $b$

(by making the photon  $b$  pass through BS and HWP<sub>2</sub>), the state  $|\Phi_b\rangle_0$  is changed to be  $|\Phi'_b\rangle_0 \equiv (\beta'_1|R\rangle + \beta'_2|L\rangle)_b \otimes (\delta'_1|b_1\rangle + \delta'_2|b_2\rangle)$ . Here  $\beta'_1 = \frac{1}{\sqrt{2}}(\beta_1 + \beta_2)$ ,  $\beta'_2 = \frac{1}{\sqrt{2}}(\beta_1 - \beta_2)$ ,  $\delta'_1 = \frac{1}{\sqrt{2}}(\delta_1 + \delta_2)$  and  $\delta'_2 = \frac{1}{\sqrt{2}}(\delta_1 - \delta_2)$ . Then we perform unitary operations on the two electron spins in QD<sub>1</sub> and QD<sub>2</sub>, which transform the states  $|\uparrow\rangle$  and  $|\downarrow\rangle$  to  $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$  and  $\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$ , respectively. Subsequently, we let the photon  $b$  pass through CPBS, HWP<sub>1</sub>, QD<sub>1</sub>, WP<sub>1</sub>, QD<sub>2</sub> and WP<sub>2</sub>. After the interaction between the photon  $b$  and the two QDs, the state of system composed of QD<sub>1</sub>, QD<sub>2</sub>, the photons  $a$  and  $b$  is changed from  $|\Psi_s\rangle_1$  to  $|\Psi_s\rangle_2$ . Here

$$\begin{aligned} |\Psi_s\rangle_1 &= |\Phi_{ae_1e_2}\rangle_2 \otimes (\beta'_1|R\rangle + \beta'_2|L\rangle)_b (\delta'_1|b_1\rangle + \delta'_2|b_2\rangle), \\ |\Psi_s\rangle_2 &= [|\uparrow\rangle_{e_1} \gamma_1 |a_1\rangle (\delta'_1|b_1\rangle + \delta'_2|b_2\rangle) + |\downarrow\rangle_{e_1} \gamma_2 |a_2\rangle \\ &\quad \otimes (\delta'_1|b_1\rangle - \delta'_2|b_2\rangle)] \otimes [|\uparrow\rangle_{e_2} \alpha_1 |R\rangle_a (\beta'_1|R\rangle \\ &\quad + \beta'_2|L\rangle)_b + |\downarrow\rangle_{e_2} \alpha_2 |L\rangle_a (\beta'_1|R\rangle - \beta'_2|L\rangle)_b]. \end{aligned} \quad (8)$$

By performing Hadamard operations on the two electron spins in QD<sub>1</sub> and QD<sub>2</sub> again after the photon  $b$  passes through WP<sub>2</sub>, the state of the complicated system becomes

$$\begin{aligned} |\Psi_s\rangle_3 &= \frac{1}{2} \{ |\uparrow\rangle_{e_1} [\gamma_1 |a_1\rangle (\delta'_1|b_1\rangle + \delta'_2|b_2\rangle) + \gamma_2 |a_2\rangle (\delta'_1|b_1\rangle \\ &\quad - \delta'_2|b_2\rangle)] + |\downarrow\rangle_{e_1} [\gamma_1 |a_1\rangle (\delta'_1|b_1\rangle + \delta'_2|b_2\rangle) \\ &\quad - \gamma_2 |a_2\rangle (\delta'_1|b_1\rangle - \delta'_2|b_2\rangle)] \} \\ &\quad \otimes \{ |\uparrow\rangle_{e_2} [\alpha_1 |R\rangle_a (\beta'_1|R\rangle + \beta'_2|L\rangle)_b + \alpha_2 |L\rangle_a (\beta'_1|R\rangle \\ &\quad - \beta'_2|L\rangle)_b] + |\downarrow\rangle_{e_2} [\alpha_1 |R\rangle_a (\beta'_1|R\rangle + \beta'_2|L\rangle)_b \\ &\quad - \alpha_2 |L\rangle_a (\beta'_1|R\rangle - \beta'_2|L\rangle)_b] \}. \end{aligned} \quad (9)$$

At last, we perform Hadamard operations on both the spatial mode and the polarization DOFs of the photon  $b$ , the state is transformed into

$$\begin{aligned} |\Psi_s\rangle_4 &= \frac{1}{2} \{ |\uparrow\rangle_{e_1} [\gamma_1 |a_1\rangle (\delta_1|b_1\rangle + \delta_2|b_2\rangle) + \gamma_2 |a_2\rangle (\delta_2|b_1\rangle \\ &\quad + \delta_1|b_2\rangle)] + |\downarrow\rangle_{e_1} [\gamma_1 |a_1\rangle (\delta_1|b_1\rangle + \delta_2|b_2\rangle) \\ &\quad - \gamma_2 |a_2\rangle (\delta_2|b_1\rangle + \delta_1|b_2\rangle)] \} \\ &\quad \otimes \{ |\uparrow\rangle_{e_2} [\alpha_1 |R\rangle_a (\beta_1|R\rangle + \beta_2|L\rangle)_b + \alpha_2 |L\rangle_a \\ &\quad (\beta_2|R\rangle + \beta_1|L\rangle)_b] + |\downarrow\rangle_{e_2} [\alpha_1 |R\rangle_a (\beta_1|R\rangle \\ &\quad + \beta_2|L\rangle)_b - \alpha_2 |L\rangle_a (\beta_2|R\rangle + \beta_1|L\rangle)_b] \}. \end{aligned} \quad (10)$$

By measuring the excess electron spin in the orthogonal basis  $\{|\uparrow\rangle, |\downarrow\rangle\}$ , the deterministic spatial-polarization hyper-CNOT gate can be constructed with feed forward.

If an auxiliary photon with the initial state  $|\varphi_i\rangle = \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)$  is put into an optical microcavity, the state of the system composed of the photon and the electron spin after reflection is changed as follows:

$$\begin{aligned} \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)|\uparrow\rangle &\rightarrow \frac{1}{\sqrt{2}}(|R\rangle + i|L\rangle)|\uparrow\rangle, \\ \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle)|\downarrow\rangle &\rightarrow \frac{1}{\sqrt{2}}(|R\rangle - i|L\rangle)|\downarrow\rangle. \end{aligned} \quad (11)$$

By detecting the auxiliary photon with the orthogonal linear polarization basis  $\{\frac{1}{\sqrt{2}}(|R\rangle + i|L\rangle), \frac{1}{\sqrt{2}}(|R\rangle - i|L\rangle)\}$ ,

the excess electron spin in QD can be read out. If the auxiliary photon is projected on  $\frac{1}{\sqrt{2}}(|R\rangle + i|L\rangle)$ , the excess electron spin is in the state  $|\uparrow\rangle$ . If the auxiliary photon is projected on  $\frac{1}{\sqrt{2}}(|R\rangle - i|L\rangle)$ , the excess electron spin is in the state  $|\downarrow\rangle$ . If we perform an addition sign change  $|a_2\rangle \rightarrow -|a_2\rangle$  on the photon  $a$  for  $|\downarrow\rangle_{e_1}$  in QD<sub>1</sub> and an addition sign change  $|L\rangle_a \rightarrow -|L\rangle_a$  on the photon  $a$  for  $|\downarrow\rangle_{e_2}$  in QD<sub>2</sub>, the state of the system composed of the photons  $a$  and  $b$  becomes

$$\begin{aligned} |\Psi_{ab}\rangle_c &= [\gamma_1 |a_1\rangle (\delta_1|b_1\rangle + \delta_2|b_2\rangle) + \gamma_2 |a_2\rangle (\delta_1|b_2\rangle \\ &\quad + \delta_2|b_1\rangle)] \otimes [\alpha_1 |R\rangle_a (\beta_1|R\rangle + \beta_2|L\rangle)_b \\ &\quad + \alpha_2 |L\rangle_a (\beta_1|L\rangle + \beta_2|R\rangle)_b]. \end{aligned} \quad (12)$$

One can see that there is a bit flip on the spatial mode of the photon  $b$  (the target qubit) when the spatial mode of the photon  $a$  (the control qubit) is  $|a_2\rangle$ . Moreover, there is a bit flip on the polarization of the photon  $b$  when the polarization of the photon  $a$  is  $|L\rangle_a$ . That is, the outcome shown in Eq.(12) is the hyper-CNOT gate operating on the two-photon system on both its polarization and its spatial-mode DOFs. Moreover, this hyper-CNOT gate works in a deterministic way in principle.

### III. PREPARATION OF TWO-PHOTON FOUR-QUBIT CLUSTER STATE

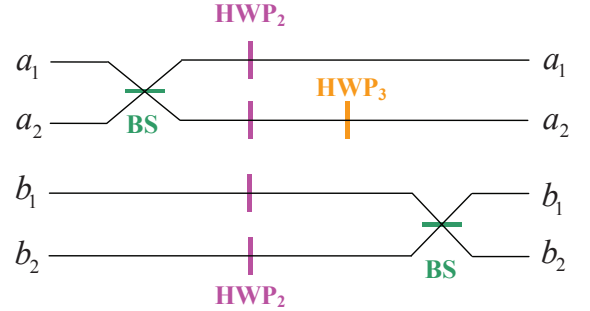


FIG. 3: (Color online) Schematic diagram for the preparation of a two-photon four-qubit cluster state with a hyperentangled Bell state. HWP<sub>2</sub> is used to perform a polarization Hadamard operation. HWP<sub>3</sub> represents a half-wave plate which is used to perform a polarization phase-flip operation  $U_p = -|R\rangle\langle R| + |L\rangle\langle L|$ . BS represents beam splitter (50:50) which is used to perform a spatial mode Hadamard operation.

As an application of our hyper-CNOT gate, we discuss the generation and the complete analysis of two-photon four-qubit cluster entangled states below. One can see that these tasks can be accomplished easily and simply with a hyper-CNOT gate and some linear-optical elements.

With our hyper-CNOT gate operating on the spatial-mode and the polarization DOFs of a photon pair, it is, in principle, easily to prepare the hyperentangled two-

photon four-qubit state

$$|\Phi_{ab}\rangle_1 = \frac{1}{2}(|R\rangle_a|R\rangle_b + |L\rangle_a|L\rangle_b) \otimes (|a_1\rangle|b_1\rangle + |a_2\rangle|b_2\rangle). \quad (13)$$

With linear optical elements, this hyperentangled state can be transformed into a two-photon four-qubit cluster state.

As shown in Fig.3, after the Hadamard operations performed on both the polarization and the spatial mode of the photon  $a$ , the hyperentangled Bell state becomes

$$|\Phi_{ab}\rangle_2 = \frac{1}{4}[(|R\rangle + |L\rangle)_a|R\rangle_b + (|R\rangle - |L\rangle)_a|L\rangle_b] \otimes [(|a_1\rangle + |a_2\rangle)|b_1\rangle + (|a_1\rangle - |a_2\rangle)|b_2\rangle]. \quad (14)$$

Then we perform the spatial mode controlled polarization phase-flip gate (HWP) on the photon  $a$ , and the two-photon state  $|\Phi_{ab}\rangle_2$  is transformed into  $|\Phi_{ab}\rangle_3$ . Here

$$\begin{aligned} |\Phi_{ab}\rangle_3 = & \frac{1}{4}\{[(|R\rangle + |L\rangle)|a_1\rangle - (|R\rangle - |L\rangle)|a_2\rangle]_a|R\rangle_b|b_1\rangle \\ & + [(|R\rangle + |L\rangle)|a_1\rangle + (|R\rangle - |L\rangle)|a_2\rangle]_a|R\rangle_b|b_2\rangle \\ & + [(|R\rangle - |L\rangle)|a_1\rangle - (|R\rangle + |L\rangle)|a_2\rangle]_a|L\rangle_b|b_1\rangle \\ & + [(|R\rangle - |L\rangle)|a_1\rangle + (|R\rangle + |L\rangle)|a_2\rangle]_a|L\rangle_b|b_2\rangle\}. \end{aligned} \quad (15)$$

After performing Hadamard operations on both the polarization and the spatial mode DOFs of the photon  $b$  at last, we get the two-photon four-qubit cluster state

$$|\Phi_{ab}\rangle_4 = \frac{1}{2}[|a_1\rangle|b_1\rangle(|R\rangle_a|R\rangle_b + |L\rangle_a|L\rangle_b) - |a_2\rangle|b_2\rangle(|R\rangle_a|R\rangle_b - |L\rangle_a|L\rangle_b)]. \quad (16)$$

From the process of the preparation of two-photon four-qubit cluster state, one can see that this state can be disentangled to the hyperentangled Bell state  $|\Phi_{ab}\rangle_1$  with some Hadamard operations and a spatial-mode controlled polarization phase-flip gate using linear optical elements. With a hyper-CNOT gate, the 16 hyperentangled Bell states can be transformed into 16 two-photon four-qubit product states which can be distinguished simply with linear optical elements and single-photon detectors. That is, the 16 hyperentangled Bell states can be analyzed easily with our spatial-polarization hyper-CNOT gate.

#### IV. DISCUSSION AND CONCLUSION

The spatial-polarization hyper-CNOT gate is constructed with the interaction of circularly polarized lights and one-side QD-cavity systems. According to Pauli's exclusion principle, this optical property is caused by the different reflection phase shifts of the left and the right circularly polarized lights. By adjusting the frequencies

as  $\omega_c = \omega_{X^-} = \omega_0$ ,  $\omega - \omega_c \approx \kappa/2$  and the cavity side leakage rate as  $\kappa_s < 1.3\kappa$  [29], the relative phase shift of circularly polarized lights can achieve  $\Delta\varphi = -\pi/2$ . Young *et al* [34] investigated the quantum-dot-induced phase shift experimentally in 2011, and showed that a QD-induced phase shift of 0.2 rad between an (effectively) empty cavity ( $Q \sim 51000$ ,  $d = 2.5\mu\text{m}$ ) and a cavity with a resonantly coupled QD can be deduced using a single-photon level probe. The Hadamard operation and unitary rotation operation of an electron spin can be completed by single-spin rotations using nanosecond electron spin resonance microwave pulses [35]. The electron spin coherence time can be extended to  $\mu\text{s}$  using spin echo techniques [29] to protect electron spin coherence with microwave pulses. The optical coherence time of exciton is ten time longer than cavity photon lifetime [36], with which optical dephasing only reduces the fidelity a few percent. The hole spin dephasing is dominant in spin dephasing of  $X^-$ , and it can be safely neglected with hole spin coherence time three orders longer than the cavity photon lifetime [37]. The heavy-light hole mixing, which causes optical selection rule imperfect, could be reduced by engineering the shape, size and type of charged exciton [29].

In an ideal condition in which one neglects the cavity side leakage, the reflection coefficients are  $|r_0(\omega)| \cong 1$  and  $|r_h(\omega)| \cong 1$  and the fidelity of our spatial-polarization hyper-CNOT gate is nearly 100%. Unfortunately, it is impossible to neglect the side leakage with cavity rigorous limitation in the QD-micropillar cavity in experiment, and the fidelity is reduced to  $F = |\langle\psi_f|\psi\rangle|^2$ , where  $|\psi\rangle$  and  $|\psi_f\rangle$  describe the final states of an ideal condition and a whole process considering external reservoirs, respectively. The Faraday rotation angle is not very sensitive to the side leakage when  $\kappa_s < \kappa$  [26]. Therefore, the fidelity  $F$  and the efficiency  $\eta$  of our spatial-polarization hyper-CNOT gate are

$$\begin{aligned} F &= \frac{[2(|r_0| + |r_h|)^2]^4}{[2(|r_0| + |r_h|)^2 + |r_0 - r_h|^2](|r_0|^2 + |r_h|^2)^2} \\ &\quad \times \frac{(|r_0| + |r_h|)^4}{(|r_0|^2 + |r_h|^2)^2}, \\ \eta &= \frac{1}{2}(|r_0|^2 + |r_h|^2)^4. \end{aligned} \quad (17)$$

Figure 4 is the fidelity and efficiency of our spatial-polarization hyper-CNOT gate, and it shows that this hyper-CNOT gate can work efficiently in strong coupling regime ( $g > (\kappa + \kappa_s)/4$ ). It is challenging to achieve strong coupling in QD-cavity system experimentally. The coupling strength can be improved from  $g \cong 0.5(\kappa + \kappa_s)$  ( $Q = 8800$ ) [38] to  $g \cong 2.4(\kappa + \kappa_s)$  ( $Q \sim 40000$ ) [39] by engineering the sample designs, growth, and fabrication in  $d = 1.5\mu\text{m}$  micropillar microcavities. If the coupling strength is  $g \cong 0.5(\kappa + \kappa_s)$ , we can get the fidelity and efficiency as  $F = 94.3\%$  and  $\eta = 48.9\%$  for  $\kappa_s/\kappa = 0$ . While if the coupling strength is  $g \cong 2.4(\kappa + \kappa_s)$ , we can get the fidelity and efficiency as  $F = 100\%$  and  $\eta = 96.3\%$  for  $\kappa_s/\kappa = 0$ , and  $F = 94.7\%$  and  $\eta = 47.3\%$  for  $\kappa_s/\kappa = 0.2$ .



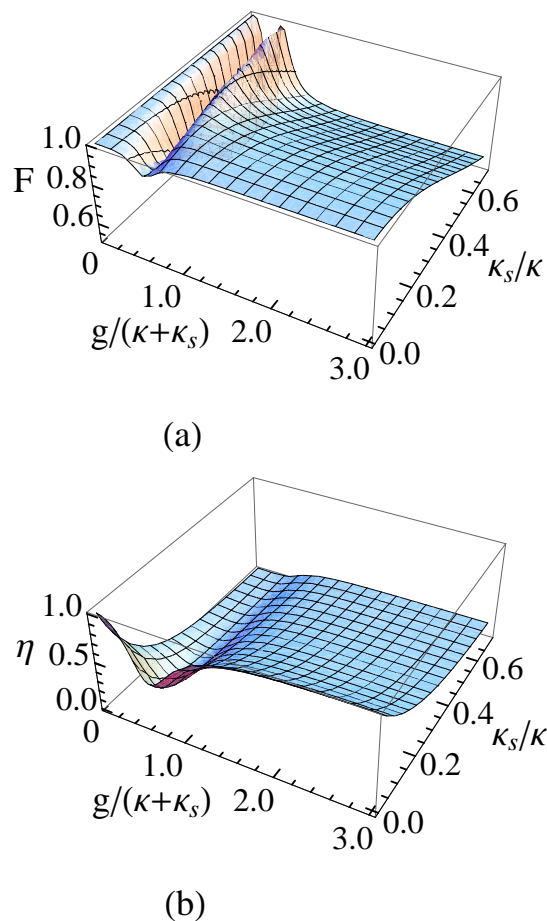


FIG. 4: (Color online) Fidelity and efficiency of spatial-polarization hyper-CNOT gate vs the coupling strength and side leakage rate with  $\gamma = 0.1\kappa$ .

Both the fidelity and efficiency become high with strong coupling strength, but they are reduced by the side leakage of cavity. The quality factor is dominated by side leakage and cavity loss rate in micropillar. Hu *et al* [29] have thinned down the top mirror to decrease the quality factor  $Q$  and side leakage and cavity loss rate  $\kappa_s/\kappa$  in high- $Q$  micropillar ( $d = 1.5\mu\text{m}$ ), and they got  $g \cong \kappa + \kappa_s$

( $Q \sim 17000$ ) and  $\kappa_s/\kappa \sim 0.7$ . In recent experiment, the strong coupling strength had achieved in large micropillar ( $d = 7.3\mu\text{m}$ ) with large side leakage [40]. It is quite demanded for high efficiency operation to observe small  $\kappa_s/\kappa$  in strong coupling regime.

In conclusion, the construction of a deterministic spatial-polarization hyper-CNOT gate can be achieved with the giant optical Faraday rotation induced by a single-electron spin in a quantum dot inside a one-side optical microcavity as a result of cavity QED. In order to obtain the giant optical Faraday rotation, the frequencies of input light and cavity mode should be adjusted to  $\omega - \omega_c \approx \kappa/2$ , and the side leakage and the cavity loss rate  $\kappa_s/\kappa$  should be controlled as small as possible. With this hyper-CNOT gate, the preparation of two-photon four-qubit cluster states is, in principle, easy and the complete analysis of hyperentangled Bell states of two-photon systems is simple. We analyzed the experimental feasibility of this hyper-CNOT gate, concluding that it can be implemented with current technology. This hyper-CNOT gate could give the powerful capability for quantum computing and quantum communication.

Certainly, we only discuss the construction of the spatial-polarization hyper-CNOT gate by exploiting the nonlinear optical property of one-side quantum dot-cavity systems. It can be in principle constructed with other systems based on nonlinearity, such as cross-Kerr media, nitrogen-vacancy centers, wave-guide nanocavity, and so on. It is possible to construct multi-photon quantum logical gates in the same way, which means a scalable quantum computing based on two DOFs of photon systems is feasible. Moreover, the present hyper-CNOT gate can be used to create multi-photon hyperentangled states in more than one DOF and complete the analysis on these multi-photon hyperentangled states as well.

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